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Heterogeneity and the earthquake magnitude-frequency distribution

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Abstract: Regional earthquake populations are widely observed to have power-law magnitude-frequency statistics. Whether earthquake distributions on individual faults are similarly power-law is presently the subject of considerable research effort. Recent careful observations suggest that event statistics on rough faults are power-law whereas earthquake distributions on smooth faults are better described by a characteristic model in which there are more large earthquakes than would be expected by extrapolation from the distribution of small events. Here the relation between heterogeneity and the magnitude-frequency distribution is investigated in a cellular automaton that includes new nearest-neighbor stress transfer rules which produce realistic stress concentrations. The results agree with the suggestion that smooth faults have more characteristic earthquake distributions and support the view that an earthquake's size is controlled by rupture arrest.

Introduction

It has long been recognized that the magnitude-frequency statistics of regional earthquake populations follow a power-law given by the Gutenberg-Richter [1944] relation

$$\log N = a - bM \qquad (1)$$

where N is the number of earthquakes of magnitude greater or equal to M, a is a constant related to the level of seismic activity, and b is the slope of the power-law. The near universality of the G-R relation has been proposed to result either from a fractal network of faults each having strictly characteristic events proportional to their fault length [*King*, 1983; *Schwartz and Coppersmith*, 1984] or from power-law earthquake distributions on individual faults [*Hanks*, 1979; *Kagan*, 1993].

Observations of earthquake populations on individual faults [*Wesnousky et al*, 1983, *Wesnousky*, 1994] tend to support a 'weak' characteristic earthquake model [*Hamilton and McCloskey*, 1997] in which events occur over a range of scales but the number of large events is greater than would be predicted by extrapolating from the power-law distribution of small earthquakes. The validity of these observations has been questioned by researchers who argue that the number of large events on any given fault is too low to be statistically significant and hence apparently characteristic earthquake distributions may simply be artifacts resulting from limited sampling of non-stationary sequences [*Wesnousky et al*, 1983; *Kagan*, 1993].

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A recent detailed study [*Stirling et al.*, 1996] of the magnitude-frequency distributions of seismicity associated with 22 individual strike slip faults strongly supports the weak characteristic earthquake model. The authors compared the observed seismicity associated with each fault (defined in the study as all events located no more than 20 km from the fault trace and normalized per unit time) to the number of large events both observed paleoseismically and estimated from geologic slip rates and concluded that the number of large events in all but 4 cases was greater than predicted by extrapolation of the small events. The authors characterized the degree of this enhancement by calculating the ratio of the number of large events observed (or estimated) to the number of large events predicted by extrapolation of the power-law trend and found that faults with greater amounts of cumulative slip had more characteristic behavior than those with less cumulative slip. They also found that faults with greater displacement were less geometrically complex than faults with limited displacement and therefore suggested that smoother faults are more likely to have characteristic earthquake distributions.

Earthquake models, however, often exhibit power-law size-frequency distributions on individual faults [*Bak and Tang*, 1989; *Carlson and Langer*, 1989; *Langer et al.*, 1996] and a number of authors have argued that these statistics arise solely from earthquake dynamics and hence even smooth models produce power-law distributions of event sizes [*Langer et al.*, 1996]. These claims are disputed by others [*Rice*, 1993; *Ben-Zion and Rice*, 1997] who insist that the algorithm used in such 'inherently discrete' models (sudden stress drop and no displacement-dependent friction) imposes an a priori heterogeneity that is the reason for the observed power-law magnitude-frequency distributions. By contrast, in smooth models with displacement-dependent friction that are discretized at scales much smaller than the critical slip weakening distance (D_c), strongly characteristic earthquake distributions with very few small events are observed [*Ben-Zion and Rice*, 1997]. The physical difference between these two classes of model is that with displacement-dependent friction discretized much smaller than D_c, a large number of cells must fail together before an instability occurs and this group of cooperatively failing cells causes stress to concentrate ahead of the rupture front.

Although a relation between heterogeneity and the earthquake magnitude-frequency distribution has been suggested from observations of natural faults [*Stirling et al*, 1996] and earthquake models [*Rundle and Klein*, 1995; *Ben-Zion and Rice*, 1997], the effect of heterogeneity on the earthquake magnitude-frequency distribution has not been systematically investigated. Ideally, the effect of heterogeneity would be studied in an elasto-dynamic model discretized at a cell size much smaller than D_c. However, as D_c measured in the laboratory is typically a few microns

[*Dieterich*, 1986] and there is presently no agreement that D_c in the field is any larger [*Ohnaka*, 1992; *Dieterich and Kilgore*, 1996], the number of cells required to model, for example, a 30 x 15 km fault is too large to be computationally feasible. Since the physical difference between inherently discrete models and models discretized much smaller than D_c is that the latter produce stress concentrations, an alternative approach is to develop an inherently discrete model that includes realistic stress concentrations. This can be accomplished in a cellular automaton by appropriate formulation of the nearest-neighbor stress transfer rules; as such models require relatively little computation time they are ideal for investigating the effects of heterogeneity.

Method

Nearest-neighbor stress transfer in standard earthquake cellular automata [e.g. *Bak and Tang*, 1989] is independent of the state of the neighbors; all neighbors receive an equal proportion of stress from a failing cell irrespective of whether they have previously failed in the ongoing event. In the simplest models, the stress transferred to each nearest-neighbor ($\Delta\sigma_{n-n}$) is given by

$$\Delta\sigma_{n-n} = \frac{\Delta\sigma}{N} \tag{2}$$

where $\Delta\sigma$ is the stress drop of the failing cell and N is the number of nearest-neighbors (4 if neighbors are defined as cells that share edges, 8 if neighbors share either edges or corners). These models are often conservative, 100% of the stress from a failing cell is transferred to its nearest-neighbors except at boundaries.

Stress concentrations can be introduced into a cellular automaton by allowing stress to be transferred only to unbroken nearest-neighbors [*Lomnitz-Adler*, 1993] such that

$$\Delta\sigma_u = \frac{\Delta\sigma}{N - n_b}$$

$$\Delta\sigma_b = 0 \tag{3}$$

where n_b is the number of previously failed nearest-neighbors, $\Delta\sigma_u$ is the stress transferred to each unbroken neighbor, and $\Delta\sigma_b$ is the stress distributed to each previously failed nearest-neighbor. This rule leads to stress concentrations that scale with the rupture size [*Steacy and McCloskey*, 1998], higher than the square root of rupture size scaling predicted from fracture mechanics [e.g. *Li*, 1987].

Here we introduce a rule in which stress from a failing cell is preferentially distributed to unbroken nearest-neighbors while previously failed cells receive a smaller proportion of stress according to

$$\Delta\sigma_u = \frac{\Delta\sigma}{(N+1) - \sqrt{(n_b + 1)}}$$

$$\Delta\sigma_b = \frac{\Delta\sigma - \sum_{i=1}^{N-n_b} \Delta\sigma_u}{n_b}, \quad n_b \neq 0 \,. \tag{4}$$

This formulation has the properties that stress is conserved (except at boundaries) and that in the limit n_b = 0, $\Delta\sigma_u = \Delta\sigma_{n-n}$. The rule leads to stress concentrations that, for a circular rupture in a homogeneous medium, initially scale with the square root of the rupture radius then asymptotically approach a constant value. Such a finite limit to the stress concentration would be expected [*Lomnitz-*

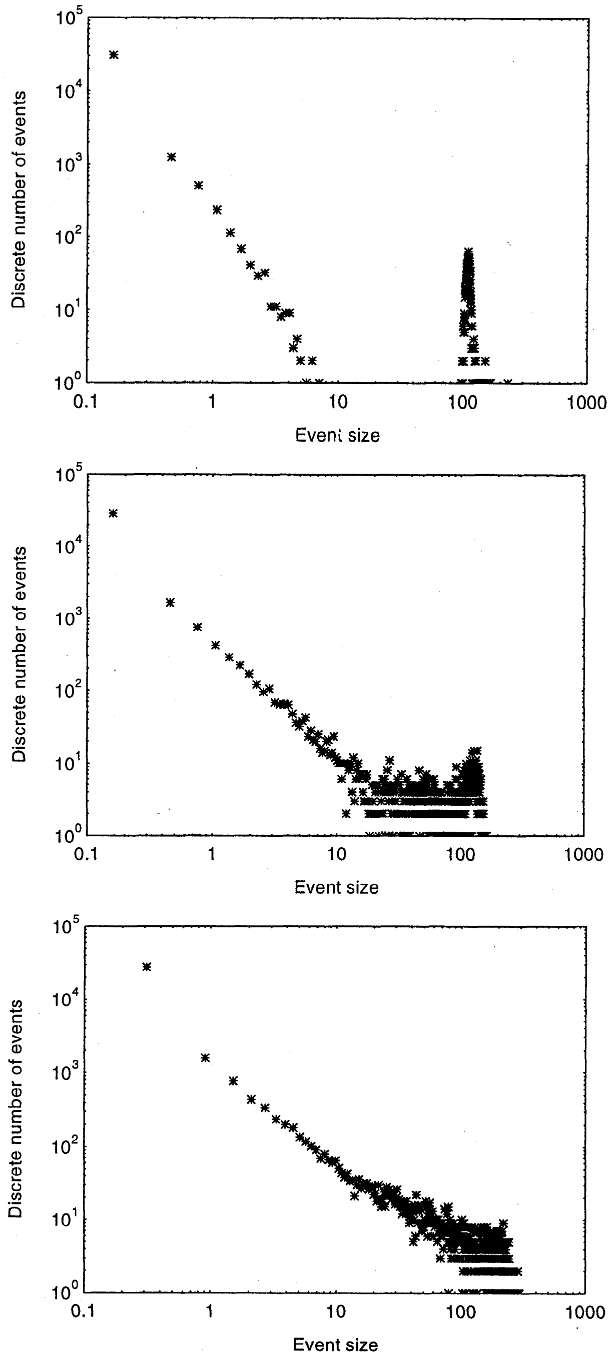

Figure 1. Discrete magnitude-frequency distributions for last 35,000 events in representative simulations. Each simulation was run on a 128x64 grid with a fractal distribution of strength (D=2.1). A) Range of strength = 0.01-0.02. Note strong enhancement of large events and absence of intermediate size ones. B) Range of strength = 0.01-0.04. Enhancement of large events is less and intermediate size ones are present. C) Range of strength = 0.01-0.10. Both degree of enhancement and slope of power-law portion of distribution have decreased.

Adler, 1992] if, as suggested by many studies, earthquakes propagate as slip pulses and only subsets of the rupture area are slipping at any one time [e.g. *Heaton*, 1990; *Wald and Heaton*, 1994].

The automaton used in this study obeys the following rules: 1) The model is loaded uniformly. 2) A cell fails when its stress exceeds its strength. 3) Stress from a failing cell is distributed according to equation (4) above (N=8). 4) The size of an event is the sum of the stress drops of all failures in response to a single increment of loading. 5) Boundaries are dissipative.

Heterogeneity is included in the model through the strength distribution. We choose a fractal distribution of strength (here D=1.6 but the results are not sensitive to the choice of fractal dimension) based on observations that fault surface roughness and gouge are fractal. The amplitude of the heterogeneity is controlled by first producing a fractal set of values between 0 and 1 and then re-normalizing them over the desired strength range. The values of strength are arbitrary but the ratio of loading rate to minimum strength is chosen so that events rarely nucleate at more than one site in response to a single increment of loading.

Results

The effect of heterogeneity on the magnitude frequency distribution is investigated by running a suite of simulations with different ranges of strength. In all cases the minimum strength is set to 0.01 and the distribution is scaled so that the maximum possible strength varies between 0.02 and 0.10. Representative magnitude-frequency distributions are shown in Figure 1 for three values of maximum strength: 0.02, 0.04, and 0.10. When the amplitude of the heterogeneity is very small (Fig. 1a) only a limited range of small events follow a (very steep) power-law distribution, intermediate size events are not observed, and the number of large events is greatly enhanced. As the amplitude of the heterogeneity increases (Fig. 1b), the degree of enhancement diminishes and the power-law portion of the distribution extends over a greater range and includes intermediate size events. Increasing the heterogeneity still further (Fig. 1c) leads to progressively less enhancement and a decrease in the slope of the power-law.

These observations are quantified in two ways. The b-value is determined by fitting a line to the power-law portion of the distribution and calculating the slope. The degree of enhancement is computed by first extrapolating the power-law portion of the distribution to its intersection with the x-axis. The number of enhanced events is simply the number of events larger than this intersection size and the degree of enhancement is the ratio of this number to the total number of events in the distribution which is held constant in the analysis. The results are shown in Figure 2. As the range of heterogeneity increases, both the b-value (Fig. 2a) and the degree of enhancement (Fig. 2b) are observed to decrease.

Discussion

Changes in magnitude-frequency distributions due to heterogeneity have also been observed in a slider-block cellular automaton. Rundle and Klein [1993] found that homogeneous systems produced power-law distributions independent of loading rate whereas heterogeneous systems produced distributions that systematically changed from sub-critical to power-law to super-critical as the loading velocity increased (see Main [1996] for a discussion). The b-values of the power-law portions of each distribution, however, remained constant. In our model, by contrast, both the degree of enhancement and the b-value vary systematically with heterogeneity; the latter has also been observed in acoustic emission experiments [*Mogi*, 1967]. In addition, our results are not sensitive to the size of the loading increment, increasing or decreasing it by an order of magnitude has little effect.

Our results can be understood by considering the interplay of dynamic stress concentrations and fault zone heterogeneity. When the amplitude of the heterogeneity is very small, the stress concentrations are sufficient to break through the stronger patches on the fault and hence events that would be stopped at intermediate size with stronger heterogeneity instead grow into much larger earthquakes. With higher heterogeneity these same events are stopped and thus the number of intermediate size events increases and the degree of enhancement decreases. The increase in the number of intermediate size events leads to the observed decrease in b-value. This latter result may be somewhat counterintuitive but the b-value only reflects the distribution of small and intermediate size events since the large events

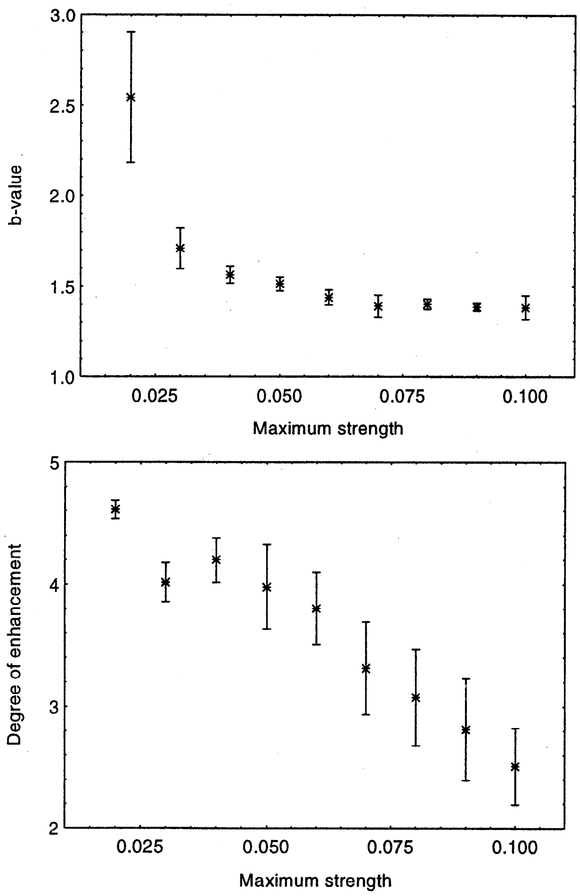

Figure 2. Dependence of magnitude-frequency distribution on amplitude of heterogeneity. Each point represents the mean of 5 realizations and error bars show standard deviation. Strength varies from 0.01 to maximum strength plotted along x-axis. A) Slope (b-value) of power-law portion of distribution. B) Degree of enhancement of large events.

belong to the enhanced part of the distribution and are not used in the b-value calculation.

The observation that heterogeneity is required to stop rupture supports both the contention [*Rice*, 1993] that power-law magnitude-frequency distributions result from heterogeneity and the view that earthquake size is controlled by rupture arrest, not rupture nucleation [*Abercrombie and Mori*, 1994; *Steacy and McCloskey*, 1998]. The results agree with the suggestion that smooth faults are more likely to have characteristic earthquake distributions than rough faults [*Stirling et al.*, 1996] and provide a physical basis to explain observations of characteristic earthquake distributions. In addition, if the statistics of earthquake magnitude-frequency distributions vary from fault to fault depending on fault zone heterogeneity, then the physics underlying the Gutenberg-Richter relation is more complex than has previously been suggested.

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